Catenary Design for Membrane Structures

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Abstract

Catenary systems are commonly used to evenly apply pre-tensioning loads on to membranes for space inflatable structures. This research was initiated by seeing some membrane structures that wrinkles were introduced by catenary systems. In the design of a catenary system, it is necessary to know the relations involving cable tension, span, sag, length of the cable, and the membrane stress introduced by the catenary system. These parameters can be determined by the static analysis of the catenary system.

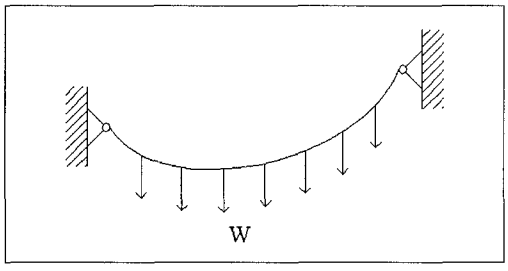

Figure 1-a. One span catenary system

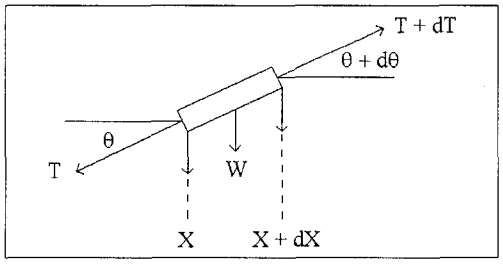

Figure 1-b. An infinitesimal element of the cable

Considering the one span catenary system in figure 1-a, in order to avoid wrinkles, the distributed load must be constant and vertical. The heights of two ends may not be the same. It is assumed that the distributed load is w units per unit horizontal length. At the location x, the tension force along the cable is T and angle between the cable and the horizontal axis is θ. At the location x+dx, the tension force is T+dT and the angle is $\theta+d\theta$. Figure 1-b represents the free-body diagram of an infinitesimal element of the cable. The equilibrium equations for both horizontal and vertical directions are given as[1]:

$$(T+dT)\sin(\theta+d\theta) = T\sin(\theta) + wdx \quad (1)$$

$$(T+dT)\cos(\theta+d\theta) = T\cos(\theta) \quad (2)$$

After some derivations, the differential equation of the flexible cable can be expressed as:

$$\frac{d^2y}{dx^2} = \frac{w}{T_0} \tag{3}$$

Where, T_0 is the horizontal cable force and is a constant number.

The curvature of the cable, the cable force, the maximum cable force and its location, the length of the cable, and other parameters will be derived by this study. Three kinds of catenary systems (catenary with different end heights, catenary with same end heights, and partially loaded catenary) will be discussed.

For the purpose of demonstrate how to use all equations derived by this study, an example of how to determine design variables of an inflatable reflectarray antenna (figure 2) will be given. Figure 3 gives analysis results for the determination of design variables.

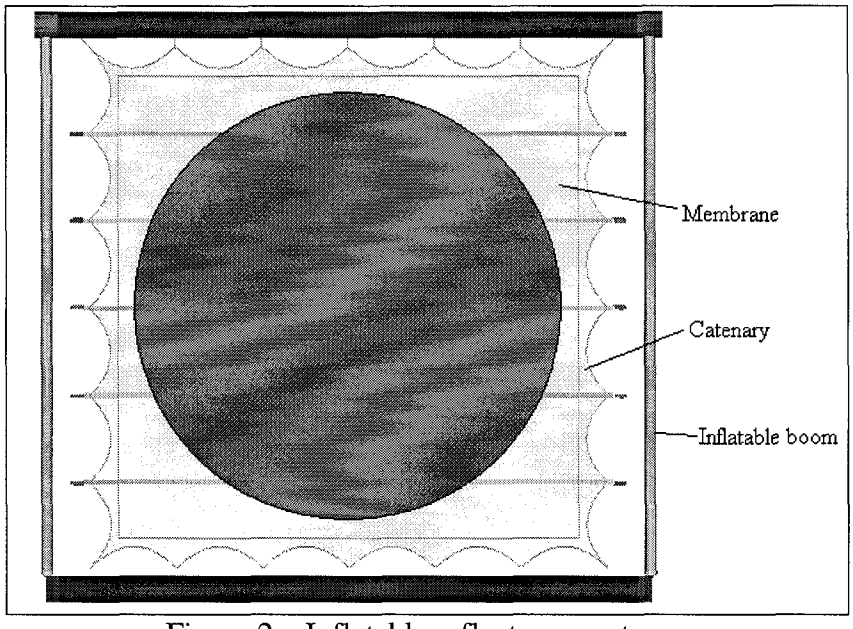

Figure 2. Inflatable reflectarray antenna

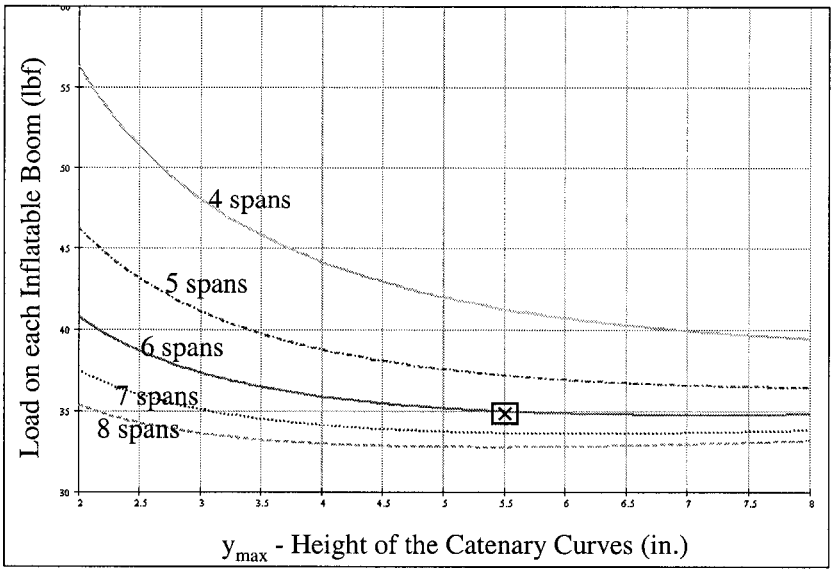

Figure 3. Catenary analysis results

In order to accomplish the designed catenary system, a catenary test bed has been established. Several new methods of attaching catenary systems to membrane structures have been developed, tested, and will be discussed by this paper. It will be seen by test results that equations derived by this study and technologies developed by this research of attaching the catenary system are very promising.

References

[1] Irvine, Max, "Cable Structures," Dover Publications, Inc., New York, 1992